The NN phase shifts in the extended quark-delocalization, color-screening model*

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An alternative method is applied to the study of nucleon-nucleon(NN) scattering phase shifts in the framework of extended quark delocalization, color-screening model(QDCSM), where the one-pion-exchange(OPE) with short-range cutoff is included.

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I. INTRODUCTION

Quantum Chromodynamics (QCD) is generally believed to be the fundamental theory of the strong interaction. High energy processes are calculable due to the asymptotic freedom of QCD. Low energy processes are however difficult to be calculated directly from QCD, especially for multiquark systems, due to the infrared confinement and complexity. At present and even in the future, the QCD-inspired model will be a useful tool to explore strong interaction physics at low energy region.

To study the baryon-baryon interaction and multiquark system, the most popular model is the constituent quark model(CQM). There are different versions of CQM with different effective degrees of freedom. The Glashow-Isgur model (GIM) [1, 2], where the effective degrees of freedom are constituent quarks and gluons, describes the properties of hadrons successfully, and has also been extended to the study of the NN interaction. The NN short-range repulsive core is successfully reproduced, but the intermediate range attraction is missing. To remedy this shortcoming, two approaches are developed. One is to invoke meson exchange again, this leads to the 'hybrid' model [3, 4, 5], another approach is to extend the GIM, this leads to the quark delocalization color screening model(QDCSM) [6, 7]. Two approaches both fit the existed experimental data of the NN interaction, but only the QDCSM explains the long standing fact that the nuclear force and molecular force are similar except the obvious differences of the energy and length scales and answers the fundamental question why does the nucleus is approximately a collection of nucleons rather than a big "quark bag". Glozman and Riska argued that the proper effective degrees of freedom of low energy QCD are constituent quarks and Goldstone bosons only and proposed the Goldstone boson exchange model [8]. It also gives a good description of the baryon spectrum but still in its infancy for the NN interaction [9]. This model will have the same drawback as the hybrid model in the understanding of the NN interaction and nuclear structure.

All of these models enjoy the phenomenological success in the description of baryon properties and even the baryon-baryon interactions [10]. The disagreement with the experimental data can generally be repaired by fine-tuning of model ingredients and parameters. However, the more the parameters, the poorer the prediction power. In this respect QDCSM employs the least adjustable parameters to explain more physics related to the NN interaction and nuclear structure. However in the quantitative fit to the NN and Hyperon-Nucleon scattering, QDCSM is not as good as the hybrid model ones up to now [6, 11, 12]. In order to explore if the QDCSM can fit the scattering data quantitatively well, a further extensive calculation is being done.

This paper reports the results of the NN phase shifts calculated with the extended QDCSM model, where one-pion-exchange (OPE) with short-range cutoff is added to the original QDCSM. Because the effective long range NN interaction of the original QDCSM has been found to be decreased too fast due to a Gaussian wave function used for the quark orbital motion and so the well established long range π exchange is missing. This extended QDCSM reproduces the deuteron properties very well [7, 13]. In this calculation the tensor part of one gluon exchange(OGE) is also taken into account. A new method of calculating the phase shift is tested. Instead of the usual asymptotic connection to the scattering wave function, we first solve the equation with a zero boundary condition as one does for the bound state problem, by varying the boundary to check if the solution is an exponential decreasing bound state or an oscillating scattering state, then compare the oscillating wavefunction with that of free particle to obtain the phase shift. This has been used in the lattice QCD calculation of the phase shift and so we check it with this otherwise can be calculated by the normal method.

This paper is organized as follows: in Sect.II. a brief introduction of QDCSM and calculation method is given, results and conclusions are given in Sect.III.

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II. HAMILTONIAN, WAVE FUNCTIONS AND CALCULATION METHOD

The details about QDCSM can be found in the Refs.[6, 14, 15]. Here only the model Hamiltonian, wave functions and the necessary equations are given. The resonating-group method (RGM), used to calculate the scattering phase shifts, can also be found in Refs.[15, 16].

In QDCSM, the Hamiltonian for the 3-quark system is the same as the usual potential model and for the 6-quark system, it is assumed to be

$$\begin{aligned}
H_6 &= \sum_{i=1}^{6}(m_i+\frac{p_i^2}{2m_i})-T_{CM}+\sum_{i<j=1}^{6}\left(V_{ij}^C+V_{ij}^G+V_{ij}^{\pi}\right), \\
V_{ij}^C &= -a_c\vec{\lambda}_i\cdot\vec{\lambda}_j\begin{cases} r_{ij}^2 & \text{if } i,j \text{ occur in the same baryon orbit,} \\ \frac{1-e^{-\mu r_{ij}^2}}{\mu} & \text{if } i,j \text{ occur in different baryon orbits,}\end{cases} \\
V_{ij}^G &= \alpha_s\frac{\vec{\lambda}_i\cdot\vec{\lambda}_j}{4}\left[\frac{1}{r_{ij}}-\frac{\pi\delta(\vec{r}_{ij})}{m_im_j}\left(1+\frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right)+\frac{1}{4m_im_j}\left(\frac{3(\vec{\sigma}_i\cdot\vec{r}_{ij})(\vec{\sigma}_j\cdot\vec{r}_{ij})}{r_{ij}^5}-\frac{\vec{\sigma}_i\cdot\vec{\sigma}_j}{r_{ij}^3}\right)\right], \qquad (1)\\
V_{ij}^{\pi} &= \theta(r_{ij}-r_0)f_{qq\pi}^2\vec{\tau}_i\cdot\vec{\tau}_j\frac{1}{r_{ij}}e^{-\mu_\pi r_{ij}} \\
&\quad\times\left[\frac{1}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j+\left(\frac{3(\vec{\sigma}_i\cdot\vec{r}_{ij})(\vec{\sigma}_j\cdot\vec{r}_{ij})}{r_{ij}^2}-\vec{\sigma}_i\cdot\vec{\sigma}_j\right)\left(\frac{1}{(\mu_\pi r_{ij})^2}+\frac{1}{\mu_\pi r_{ij}}+\frac{1}{3}\right)\right],
\end{aligned}$$

$$\theta(r_{ij}-r_0) = \begin{cases} 0 & r_{ij}<r_0, \\ 1 & \text{otherwise,}\end{cases}$$

where all the symbols have their usual meaning. OPE potential and the tensor part of OGE (which is omitted before) have been added. The OPE was introduced to account for the long range tail of NN interaction and the tensor part of OGE and OPE were included to do the S-D channel coupling. In the OPE potential, $\theta(r)$ is the cutoff function, which is introduced to avoid double counting in the short range part. The cutoff parameter r_0 and color screening parameter μ are determined by the properties of deuteron. The quark-pion coupling constant $f_{qq\pi}$ can be obtained from the nucleon-pion coupling constant $f_{NN\pi}$, which is determined by experiment.

Combining RGM and generating coordinates formalism, the ansatz for the 6-quark system wave function can be written as [15, 16]

$$\begin{aligned}
\Psi_{6q} &= \mathcal{A}\sum_k\sum_{i=1}^{n}\sum_{L_k=0,2}C_{k,i,L_k}\int\frac{d\Omega_{S_i}}{\sqrt{4\pi}}\prod_{\alpha=1}^{3}\psi_\alpha(\vec{S}_i,\epsilon)\prod_{\beta=4}^{6}\psi_\beta(-\vec{S}_i,\epsilon) \\
&\quad\left[[\eta_{I_{1k}s_{1k}}(B_{1k})\eta_{I_{2k}s_{2k}}(B_{2k})]^{Is_k}Y^{L_k}(\hat{\vec{s}}_i)\right]^J[\chi_c(B_1)\chi_c(B_2)]^{[\sigma]}, \qquad (2)
\end{aligned}$$

where k is the channel index, $S_i, i=1,\cdots,n$ are generating coordinates. For example, for the deuteron bound state calculation, we have $k=1,\ldots,5$, corresponding to the channels NN $S=1$ $L=0$, $\Delta\Delta$ $S=1$ $L=0$, $\Delta\Delta$ $S=3$ $L=2$, NN $S=1$ $L=2$, and $\Delta\Delta$ $S=1$ $L=2$. ψ's are the delocalized quark orbital wave functions.

$$\begin{aligned}
\psi_\alpha(\vec{S}_i,\epsilon) &= \left(\phi_\alpha(\vec{S}_i)+\epsilon\phi_\alpha(-\vec{S}_i)\right)/N(\epsilon), \\
\psi_\beta(-\vec{S}_i,\epsilon) &= \left(\phi_\beta(-\vec{S}_i)+\epsilon\phi_\beta(\vec{S}_i)\right)/N(\epsilon), \\
N(\epsilon) &= \sqrt{1+\epsilon^2+2\epsilon e^{-S_i^2/4b^2}}. \qquad (3)\\
\phi_\alpha(\vec{S}_i) &= \left(\frac{1}{\pi b^2}\right)^{3/4}e^{-\frac{1}{2b^2}(\vec{r}_\alpha-\vec{S}_i/2)^2}
\end{aligned}$$

$$\phi_\beta(-\vec{S}_i) = \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2}(\vec{r}_\beta+\vec{S}_i/2)^2}.$$

η and χ are the spin-isospin and color wave functions. The delocalization parameter ϵ is variationally determined by the six-quark dynamics.

With the above ansatz, the RGM equation becomes an algebraic equation,

$$\sum_{j,k,L_k} C_{j,k,L_k} H_{i,j}^{k',L'_{k'},k,L_k} = E \sum_j C_{j,k,L_k} N_{i,j}^{k',L'_{k'}} \quad (4)$$

where $N_{i,j}^{k',L'_{k'}}$, $H_{i,j}^{k,L_k,k',L'_{k'}}$ are the (eq.(2)) wave function overlaps and Hamiltonian matrix elements, respectively. By solving the generalized eigen value problem, we obtain the eigen energies of the 6-quark systems and the corresponding wave functions.

It is well known that the scattering wavefunction of the two-body scattering process is the spherical Bessel function in the asymptotic region (if there is no Coulomb interaction or it is neglected) but has an additional phase shift in comparing to the free particle ones. So the eigen wavefunction obtained from the eigen value problem (eq.(4)) of the scattering state should be proportional to the spherical Bessel function with a phase shift outside the interaction range. To fix the phase shift, we compare the nodes of the obtained wavefunction with the roots of $j_L(kR+\delta_L)=0$. In this way, a set of linear algebraic equations are obtained. The scattering energy-momentum and phase shift then can be obtained by solving these equations. In practical calculation, a boundary condition with finite range (finite R) is always used, so the boundary effect should be taken into consideration when pick up nodes.

Comparing to the conventional method in obtaining the phase shift, the new method usually takes more computing time, and doesn't work well for very low energy scattering because there are not enough nodes in the obtained wavefunctions. However the new methodhas the advantage that it does not need to set up an asymptotic boundary, where the matching conditions are used to find the phase shift. Especially for those cases, where to really get an asymptotic solution is practically too difficult and the conventional method cannot be used. The typical case is the lattice QCD calculation, where one always deals with the problem in a limited volume. This new method at least is the one to tackle the scattering for lattice QCD. It is also a convenient method to calculate the bound and scattering channel coupling problem.

III. RESULTS AND CONCLUSION

Most of the parameters in the QDCSM have been fixed by matching baryon properties, except for color screening parameter μ and the short-range cutoff r_0. In this calculations, we test three values of r_0: 0.6 fm, 0.8fm and 1.0 fm. For each cutoff, μ is determined by matching

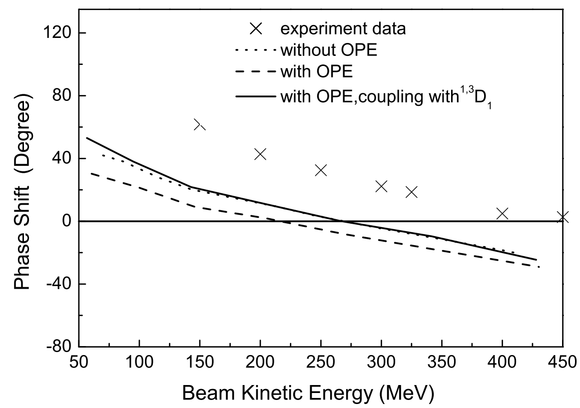

FIG. 1: $N-N$ phase shifts versus laboratory kinetic energy for the channel of $^{1,3}S_1$.

the mass of the deuteron. To save space, here we only report the results for the best one set ($r_0 = 0.8$fm) [13] For comparison, the results without OPE are also given. The model parameters aregiven in Table I.

Table I. Model parameters

	without OPE	with OPE
r_0 (fm)		0.8
m (MeV)	313	313
b(fm)	0.6034	0.6015
α_s	1.543	1.558
a_c (MeV fm^{-2})	25.132	25.135
μ (fm^{-2})	1.0	0.85
μ_π (MeV)		138.04

The following NN phase shifts, $^{3,1}S_0$, $^{3,1}D_2$, $^{1,3}S_1$, $^{1,3}D_1$, $^{1,3}D_3$, have been calculated. The symbol $^{2I+1,2S+1}L_J$ is used in the paper and all the experimental data of NN phase shifts are taken from Ref. [17, 18]. For the case of $^{1,3}S_1$, $^{1,3}D_1$, coupling between S- and D-wave is considered.

For $^{1,3}S_1$ (FIG.1), the phase shifts in the single channel calculation decrease when OPE is added due to the smaller μ used. OPE provided additional attraction and stronger S-D coupling, the color screening μ has to be reduced for compensation to reproduce the deuteron binding energy. Taking into account the effect of S-D mixing, the phase shifts rise and the results approach the experiment data again. The remaining discrepancy might be resolved by additional channel coupling and this has happened in the bound sate calculation of deuteron.

The $^{3,1}S_0$ channel has a similar results which is not given to save the space.

For the case of $^{1,3}D_1$ (FIG.2), OPE improves the phase shifts, especially when S-D mixing is considered. The S-D mixing pushes the phase shifts down, so does the

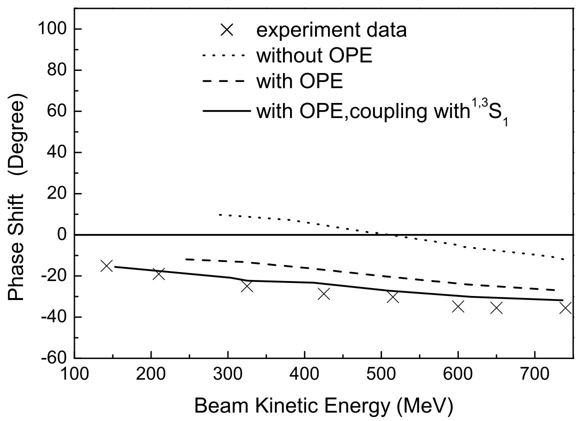

FIG. 2: Same as FIG.1 for the channel of $^{1,3}D_1$.

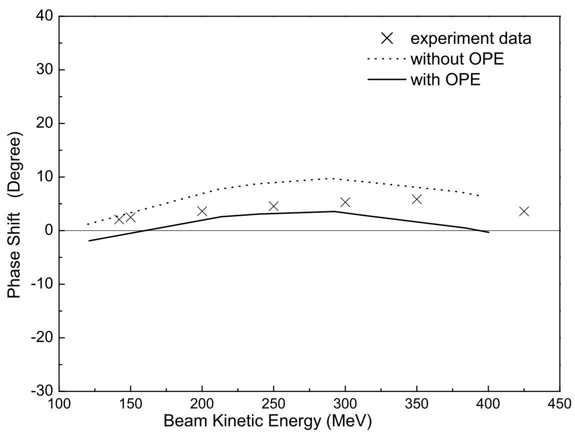

FIG. 3: Same as FIG.1 for the channel of $^{1,3}D_3$.

decreasing of color screening parameter due to the additional attraction contributed by OPE. The total effect makes the results match the experimental data very well.

As for $^{1,3}D_3$ (FIG.3), the results match the experimental data as well. As in the $^{1,3}D_1$ case, the calculated phase shifts go down to the experimental data due to the decreasing ofthe color screening parameter after adding the OPE.

These results show that QDCSM is possible to describe the NN scattering data quantitatively by including the long range OPE and channel coupling. Taking into account the following two results together, that the deuteron properties have been reproduced very well by the same extended QDCSM [7, 13] and the effective baryon-baryon interactions of the three constituent quark models have been shown to be similar[10], it is fair to conclude that the short and intermediate range NN interactions, which were attributed to the σ (or two π) and heavier meson (ρ, ω, etc.) exchanges in the Goldstone boson exchange and hybrid model, are successfully described by the quark delocalization and color screening effects.

Quark delocalization is exactly the same as the electron delocalization which is responsible for the molecular bond. The nuclear force and molecular force share the same delocalization mechanism and therefore (at least in our opinion) appear to be similar. As for the color screening, it is a phenomenology to describe what has been missing in the two body confinement. Some nonlinear QCD interactions, such as the three gluon interaction, three body instanton interaction[19], certainly can not be included in the two body confinement. However a real QCD verification of the model Hamiltonian assumed in QDCSM is still an expectation.

For the quantitative fit to the scattering data, there are still disagreement remains to be resolved in QDCSM. The spin-obit interaction of the effective one gluon exchange should be added to understand the P-wave and higher partial wave phase shifts. The $N\Lambda$, $N\Sigma$ scattering should be reanalysed with the Extended QDCSM. The $N\Xi$ scattering should be predicted before the experimental measurement and it will be a good check of different constituent quark models.

Our experience on the new method of calculating the phase shifts is preliminary. Further quantitative check with the usual method within larger energy region and more channel coupling is needed. Our message to the lattice QCD phase shift calculation is, it is a unified method to deal with the bound state and scattering but should be used with caution especially in the very low energy region and quantitative reliable results being expected.

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